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Direct Geometry And Kinematics Of Parallel Manipulators

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Abstract

A current trend in the development of products represents the commercial programs use of mathematical calculation and design. As an example in this regard, the purpose of our paper is the calculation of the direct geometry of a parallel manipulator plan, using specialized software. The application refers to a parallel plan manipulator whose mobile platform is a triangular plate, connected to a fixed platform by three pneumatic actuators. Each actuator is materialized by a chain-type kinematic RTR, which consists in two parts connected by prismatic pairs. Exterior joints of the two elements are revolute pairs. The authors of the paper propose an algorithm for computing using MATHEMATICA, to determine the coordinates of the vertices of three triangular mobile platforms. To provide start values in the iterative calculus modeling software platform SOLIDWORKS is necessary.

1. THE DIRECT GEOMETRY OF THE PLAN PARALLEL MANIPULATOR (MPPP)

The manipulator is made of a mobile platform, which is a triangular plate $B_1B_2B_3$, linked to a fixed platform $O_1O_2O_3$ by three pneumatic actuators. Each actuator is materialized by a chain-type kinematics RTR (Fig. 1) consists of two elements (beams). They have at one of the ends (O_i , B_i respectively) a half revolute pair, and at the other end (A_i) a prismatic pair.

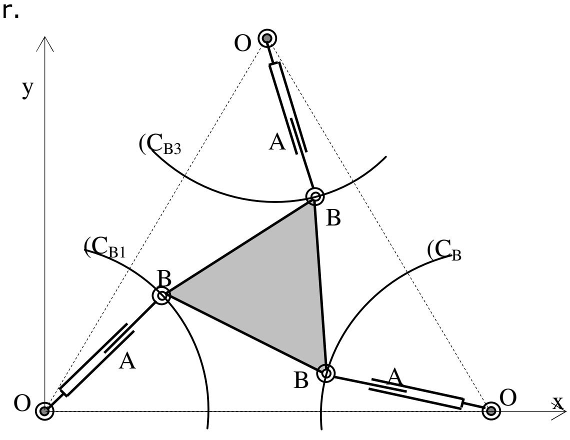

Figure 1: Plan parallel manipulator

Equations circles (CBi), are:

$(x_{Bi}-x_{Oi})^2+(y_{Bi}-y_{Oi})^2=s_i^2$, $i=1,2,3$, where i = 1,2,3 (1, 2, 3)

In this equation, s_i stands for the variable length rays of the three centres in circles with fixed points O_i. The mobility of the plan manipulator is 3, which corresponds to the length and the three parameters that are independent.

The distances between the mobile points B_1, B_2 and B_3 (triangle peaks platform) are constant known lengths:

$$\overline{B_1B_2}=l_{12};\ \overline{B_2B_3}=l_{23};\ \overline{B_1B_3}=l_{13}.$$

These constraints are presented in the following three equations:

$$(x_{B2}-x_{B1})^2+(y_{B2}-y_{B1})^2=l_{12}^2; \quad (4)$$

$$(x_{B3}-x_{B2})^2+(y_{B3}-y_{B2})^2=l_{23}^2; \quad (5)$$

$$(x_{B3}-x_{B1})^2+(y_{B3}-y_{B1})^2=l_{13}^2. \quad (6)$$

If we notate x_{Bi} with X_i, y_{Bi} with Y_i, x_{Oi} with x_i and y_{Oi} with y_i the six scalar above the equations become:

$$\begin{cases}(X_1-x_1)^2+(Y_1-y_1)^2=s_1^2;\\(X_2-x_2)^2+(Y_2-y_2)^2=s_2^2;\\(X_3-x_3)^2+(Y_3-y_3)^2=s_3^2;\\(X_2-X_1)^2+(Y_2-Y_1)^2=a^2;\\(X_3-X_2)^2+(Y_3-Y_2)^2=b^2;\\(X_3-X_1)^2+(Y_3-Y_1)^2=c^2.\end{cases} \quad (7)$$

The right terms of system equations (7) and the coordinates of fixed revolute pairs are known (Fig.1):

$$x_1=0,\ y_1=0;\ x_2=\overline{O_1O_2}=a_1,\ y_2=0;\ x_3=a_2,\ y_3=a_3.$$

The unknown terms of the system nonlinear equations (7) are: $X_1, X_2, X_3, Y_1, Y_2, Y_3$.

2. THE NUMERICAL MODELING USING THE MATHEMATICA AND SOLIDWORKS SOFTWARE

The known terms are:

$a=l_{12}=30$ mm, $b=l_{23}=30$ mm, $c=l_{13}=30$ mm, $s_1=s_2=s_3=30$ mm

$a_1=80$ mm, $a_2=40$ mm, $a_3=68$ mm

The unknown terms X1, X2, X3, Y1, Y2, Y3 can be determined by using the function FindRoot […] in the Mathematica software. In order to determine the unknown term faster and more precisely we must find the start values in the iterative calculus for the function FindRoot […] and this can be done using the computer aided design SolidWorks.

The platform is fixes in SolidWorks (Fig.1) and coordinates of points B1, B2, B3 are determined approximately, from the drawing. These represent the start values in the iterative calculus.

Then one uses the following syntax in the program Mathematica in order to find accurate solutions:

```
FindRoot[{X1^2 + Y1^2 == s1^2, (X2 - a1)^2 + Y2^2 == s2^2, (X3 - a2)^2 + (Y3 - a3)^2 == s3^2,
   (X2 - X1)^2 + (Y2 - Y1)^2 == a^2, (X3 - X2)^2 + (Y3 - Y2)^2 == b^2, (X3 - X1)^2 + (Y3 - Y1)^2 == c^2},
  {X1, 0}, {X2, 60}, {X3, 45}, {Y1, 10}, {Y2, 10}, {Y3, 30}] /.
 {s1 → 30, s2 → 30, s3 → 30, a → 30, b → 30, c → 30, a1 → 80, a2 → 40, a3 → 68}
```

One obtained the solution:

Coordinates / Point	X_i	Y_i
B_1	23,1308	19,1041
B_2	51,4126	9,09728
B_3	45,9379	38,5935

Check:

```
{X1^2 + Y1^2 == s1^2, (X2 - a1)^2 + Y2^2 == s2^2, (X3 - a2)^2 + (Y3 - a3)^2 == s3^2,
   (X2 - X1)^2 + (Y2 - Y1)^2 == a^2, (X3 - X2)^2 + (Y3 - Y2)^2 == b^2, (X3 - X1)^2 + (Y3 - Y1)^2 == c^2} /.
  {X1 → 23.130754300167318`, X2 → 51.412600110535436`, X3 → 45.93787003780497`,
   Y1 → 19.104141057023874`, Y2 → 9.09728352761694`, Y3 → 38.59350922975893`} /.
 {a1 → 80, a2 → 40, a3 → 68}

{900. == s1^2, 900. == s2^2, 900. == s3^2, 900. == a^2, 900. == b^2, 900. == c^2}
```

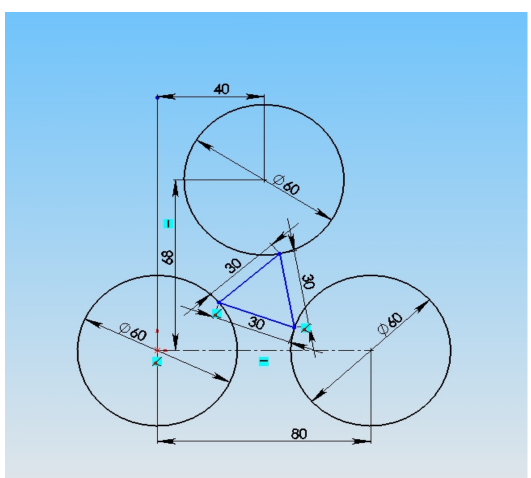

Figure 2: First solution graphical representation

For other start values:

```
FindRoot[{X1^2 + Y1^2 == s1^2, (X2 - a1)^2 + Y2^2 == s2^2, (X3 - a2)^2 + (Y3 - a3)^2 == s3^2,
   (X2 - X1)^2 + (Y2 - Y1)^2 == a^2, (X3 - X2)^2 + (Y3 - Y2)^2 == b^2, (X3 - X1)^2 + (Y3 - Y1)^2 == c^2},
  {X1, 30}, {X2, 55}, {X3, 36}, {Y1, 12}, {Y2, 16}, {Y3, 39}] /.
 {s1 → 30, s2 → 30, s3 → 30, a → 30, b → 30, c → 30, a1 → 80, a2 → 40, a3 → 68}
```

One obtained the solution

Coordinates / Point	X_i	Y_i
B_1	28,5874	9,09728
B_2	56,8692	19,1041
B_3	34,0621	38,5935

Check:

```
{X1^2 + Y1^2 == s1^2, (X2 - a1)^2 + Y2^2 == s2^2, (X3 - a2)^2 + (Y3 - a3)^2 == s3^2,
   (X2 - X1)^2 + (Y2 - Y1)^2 == a^2, (X3 - X2)^2 + (Y3 - Y2)^2 == b^2, (X3 - X1)^2 + (Y3 - Y1)^2 == c^2} /.
  {X1 → 28.587399888792547`, X2 → 56.869245699040015`, X3 → 34.06212996349518`,
   Y1 → 9.097283528519082`, Y2 → 19.104141055504737`, Y3 → 38.593509229600706`} /.
 {a1 → 80, a2 → 40, a3 → 68}

{900. == s1^2, 900. == s2^2, 900. == s3^2, 900. == a^2, 900. == b^2, 900. == c^2}
```

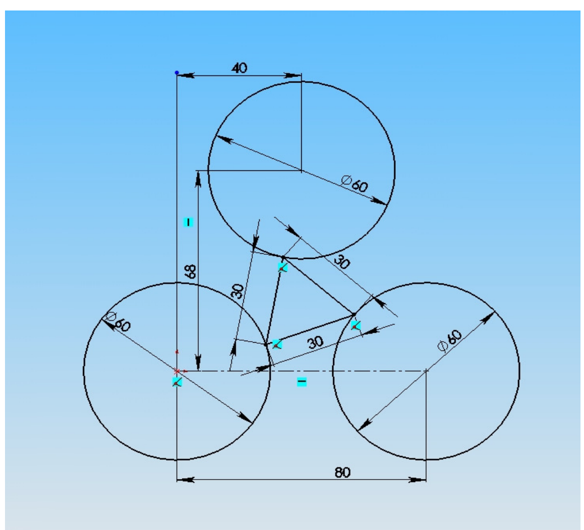

Figure 3: Second solution graphical representation

Conclusions:

1. The two syntaxes differ by the start values in the iterative calculus, around which the program will find the solution. These values are:

For the first solution: $\{X_1, 0\}, \{X_2, 60\}, \{X_3, 45\}, \{Y_1, 10\}, \{Y_2, 10\}, \{Y_3, 30\}$

For the second solution: $\{X_1, 30\}, \{X_2, 55\}, \{X_3, 36\}, \{Y_1, 12\}, \{Y_2, 16\}, \{Y_3, 39\}$.

2. Even if other start values are indicated to it, the program will no longer find solutions.

3. The values around which the solution is to be looked for must be indicated and, in many applications, these values are very difficult to intuit. Therefore, most of the times it is necessary to use a design software, which allows both the graphical representation of the manipulator, in order to guess the start values, and the graphical representation of the obtained solutions.

NOTE

[1] Antonescu, O. and P. Antonescu. "Contributions to topologic structural synthesis of the parallel manipulators", *Conference of Robotics, Iasi, Romania*, LII (LVI), Fascic 7A, 2006, pp. 23-30.

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